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The role of topology on the dynamics of the Naming Game

A. Baronchelli[1], L. Dall'Asta[2,3], A. Barrat[2,4], and V. Loreto[1]

[1] Dipartimento di Fisica, Università "La Sapienza" and SMC- INFM-CNR, P. le A. Moro 2, 00185 Roma, Italy
[2] Laboratoire de Physique Théorique (UMR du CNRS 8627), Bâtiment 210, Université de Paris-Sud, 91405 Orsay Cedex, France
[3] Abdus Salam International Center for Theoretical Physics, Strada Costiera 11, 34100 Trieste, Italy
[4] Complex Networks Lagrange Laboratory, ISI Foundation, Turin, Italy

Abstract. The Naming Game captures the essential features leading a population to agree on the use of a semiotic convention (or, more in general, an opinion). Consensus emerges through local *negotiations* between pairs of agents, in the absence of any central co-ordination. Thus, it is natural that topology, identifying the set of possible interactions, plays a central role in the dynamics of the model. Here, we review the role of different topological properties, pointing out that finite connectivity, combined with the small-world property, ensure the best performances in terms of memory usage and time to reach convergence.

1 Introduction

The recent past has witnessed an important development of the activities of statistical physicists in the area of social sciences (for a recent collection of papers see [1]). In particular, many efforts have been made to understand the mechanisms of social interactions and the resulting macroscopic phenomena [2]. Here, an interesting problem consists in determining whether a population of individuals reaches a consensus state and, in case, which process leads to such a situation. This subject is usually called *opinion dynamics*, but it describes as well very different processes, such as those concerning the emergence of language. We have recently proposed a model called (minimal) *naming game* (NG) [3], originally introduced in the field of robotics [4], that allows to study the self-organized mechanism leading to the emergence of a linguistic convention or a communication system in a population of agents. The major novelty of the naming game with respect to other models of social interactions, such as the Voter model [5], is that the usual imitation process, in which an agent takes the state of a neighbor, is replaced by a two-step negotiation process, in which memory effects play a central role.

2 The model

Let us consider a population of N identical agents on the vertices of a generic undirected graph: each agent disposes of an internal inventory, in which an a priori unlimited number of states (or *words*) can be stored. As initial conditions we require all inventories to be empty. At each time step, a pair of neighboring agents is chosen randomly, one playing as "speaker", the other as "hearer", and negotiate according to the following rules:

- the speaker selects randomly one of its words (or invents a new word if its inventory is empty) and conveys it to the hearer;

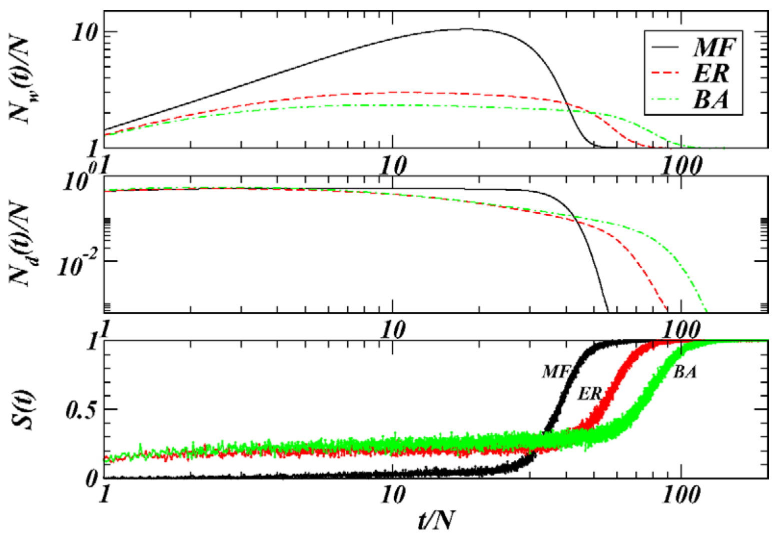

Fig. 1. Dynamics of the Naming Game on different topologies: complete graph (mean-field case - MF), Erdös Renyi (ER) random graph and Barabàsi-Albert (BA) graph, both with average connectivity $\langle k \rangle = 10$. In all cases, the process begins with an initial spreading of the different words (whose total number is $N_d(t)$), followed by a longer period in which they are exchanged among the agents. Thus, the total number of words, $N_w(t)$, grows till a maximum and then start decreasing due to successful interactions which eventually lead the system to converge ($N_d(t) = 1$, $N_w(t) = N$). In networks, finite connectivity allows for a faster initial growth in the success rate, $S(t)$, but the small-world property gives rise to the same exponential convergence observed in the fully connected graph. Data refer to populations of $N = 1000$ agents.

- if the hearer's inventory contains such a word, the two agents update their inventories so as to keep only the word involved in the interaction (*success*);
- otherwise, the hearer adds the word to those already stored in its inventory (*failure*).

Similar cooperative learning processes are present on the Web (e.g. in the case of tagging systems [6]) and may be used in sensor networks [7].

The non-equilibrium dynamics of the system is characterized by three temporal regions: (1) initially the words are invented; (2) then they spread throughout the system inducing a reorganization process of the inventories; (3) this process eventually triggers the final convergence towards the global consensus (all agents possess the same unique word). This final configuration is always reached for finite systems. However, the dynamical process leading to the final homogeneous configuration depends strongly on the topological properties of the underlying graph.

3 The role of topology

The essential quantities to describe the convergence process are the total number of words in the system, $N_w(t)$, the number of different words, $N_d(t)$, and the success rate, $S(t)$, defined as the probability of a successful interaction at a given time. For instance, on the complete graph, the process starts with an initial spreading of words (linear with time) throughout the system followed by a longer period ($O(N^{1.5})$) in which words are exchanged among the agents (see Fig. 1). After the peak, the total number of words decreases triggering a super-exponential convergence process that leads the population to the adsorbing configuration [3]. On low-dimensional lattices and hierarchical structures, on the other hand, the model converges very slowly, and the reason is related to the formation of many different local clusters of agents with the same unique word, that grow by means of coarsening dynamics [8]. Therefore in a d-dimensional lattice, the maximum memory per agent is finite (while it scales as $\sqrt{N}$ for the

Table 1. Scaling with the system size N of the maximum number of words (memory) and time of convergence. Networks, thanks to the small-world property and the finite connectivity, ensure a trade-off between the fast convergence of mean-field topology and the small memory requirements of lattices.

	Mean-field	Lattices ($d \leq 4$)	Networks
Maximum memory	$N^{1.5}$	N	N
Convergence time	$N^{1.5}$	$N^{1+\frac{2}{d}}$	$N^{1.4\pm0.1}$

complete graph), but the consensus is reached in a time $t_{conv}/N \sim N^{2/d}$, since the average cluster size grows as $t^{1/2}$.

In order to optimize the learning process, topologies realizing a trade-off between these two opposite behaviors are required. In Figure 1, the curves relative to Erdös-Renyi (ER) homogeneous random graphs [9] and Barabàsi-Albert (BA) heterogeneous networks [10] are reported. The convergence time for the NG on these topologies is very close (except for logarithmic corrections) to the mean-field one, whereas the required memory stays finite (it does not depend on N only on the average degree). In Ref. [11,12], we showed both numerically and analytically that the trade-off is ensured by the small-world properties of these networks [13].

In conclusion, the naming game is a non-equilibrium process whose properties strongly depend on the topology of the system. As summarized in Table 1, low connectivity ensures a finite requirement of memory to the single agents, while the small-world property is responsible for fast (mean-field like) convergence. Moreover, while we have focused here only on the global properties of the NG, it has also been shown that topology strongly influences the microscopic activity of single agents [14]. Before concluding, it is interesting to note that real social networks, like Web communities, have small-world properties, thus they should also present optimized self-organizing learning processes. On the other hand, our study suggests that small-world networks provide the most favorable topologies to optimize the learning time in sensor networks [7].

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